Solving with Basic Identities (3.10, 3.11, 3.12) HW

Hi B Day.

Sorry I cannot be with you today. I want you to practice solving equations and applying identities a little more. Before you do, a recap:

If you see $\sin^2\theta$, $\cos^2\theta$, $\tan^2\theta$, $\sec^2\theta$, $\csc^2\theta$, or $\cot^2\theta$, you should think "I wonder if I should apply a Pythagorean Identity to help with the problem…"

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

If you do not see anything squared but think you are still supposed to rewrite the problem differently, try applying a "reciprocal identity". Replace one function with the reciprocal of another function:

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sin\theta = \frac{1}{\csc\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

If you see $\tan\theta$ or $\cot\theta$, you can apply what we call a "quotient identity":

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

If you look at a problem and think "I know I need to rewrite this somehow, but I don't know what to try", try using the reciprocal identities and quotient identities to convert everything to $\sin\theta$ and $\cos\theta$.

For example, $\sec\theta\cot\theta + 2\csc\theta$ can be rewritten as $\left(\frac{1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) + 2\left(\frac{1}{\sin\theta}\right)$

If while solving an equation you reach a point where you only have one trig function, try replacing that function with a variable to make it less scary looking.

For example, if you reach $2\cos\theta + \frac{1}{\cos\theta} = -3$

Try rewriting as $2A + \frac{1}{A} = -3$

You can multiply both sides by A (making $2A^2 + 1 = -3A$), solve the quadratic ($A = -\frac{1}{2}$ or $A = -1$), then switch back to cosine ($\cos\theta = -\frac{1}{2}$ or $\cos\theta = -1$)

With these approaches, everything on the next page is able to be done. I'm not saying they are all easy, but I am saying I have full confidence you're ability to do hard things. Give them a try. Ask each other for help. Offer help to each other. Learn. Grow. Thrive. Get rich. Visit school fifteen years from now when you're rich and find your old AP Precalc teacher and say "now that I'm rich, I'm here to share my wealth with the teacher who set me down this path of success. Mr. QED, please accept this check for one bajillion dollars."

Solve over the interval $0 \leq \theta < 2\pi$. Be careful of extraneous solutions that are outside the domain of these functions.

1. $2\cos\theta\tan\theta = -\sqrt{2}$

2. $2\sin\theta + 5 = 3\csc\theta$

Solve over the interval $-\pi \leq \theta < \pi$. Be careful of extraneous solutions that are outside the domain of these functions.

3. $\cos\theta - 2\sec\theta = 1$

4. $\frac{\tan\theta}{\csc\theta} = \sin\theta$

Find an expression or expressions that represent ALL solutions to each equation.

5. $\frac{\tan\theta}{\sec^2\theta - 1} = \sqrt{3}$

6. $(\csc\theta - 1)(\csc\theta + 1) = 3$

7. $2\cos\theta\cot\theta + \cot\theta = \csc\theta$

8. $\cos\theta - \tan\theta\sin\theta = 0$

In each question, four expressions are presented. Three of the expressions are equivalent where defined. One is not. Find the one expression that is not equivalent to the others.

9.	$\cos^4\theta - \sin^4\theta$	$\cos^2\theta - \sin^2\theta$	$2\sin^2\theta - 1$	$2\cos^2\theta - 1$
10.	$\dfrac{\cos\theta}{\tan\theta - \sec\theta}$	$\dfrac{(1+\sin\theta)(1-\sin\theta)}{\sin\theta}$	$\csc\theta - \sin\theta$	$\dfrac{\cos^2\theta}{\sin\theta}$
11.	$\sin^2\theta - \cos^2\theta$	$\tan^2\theta + \cot^2\theta + 2$	$\sec^2\theta + \csc^2\theta$	$(\tan\theta + \cot\theta)^2$
12.	$\dfrac{\tan\theta}{1+\tan^2\theta}$	$\cos^2\theta - \sin^2\theta$	$\dfrac{\cos\theta}{\csc\theta}$	$\sin\theta\cos\theta$

Solving Basic Identities HW Answers

1. $\theta = \frac{5\pi}{4}$ or $\frac{7\pi}{4}$
2. $\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$
3. $\theta = -\pi$
4. $\theta = -\frac{3\pi}{4}$ or $\frac{\pi}{4}$
5. $\theta = \frac{\pi}{6} + 2\pi k$ or $\theta = \frac{7\pi}{6} + 2\pi k$, where $k \in \mathbb{Z}$
   This may be written more concisely as $\theta = \frac{\pi}{6} + \pi k$, where $k \in \mathbb{Z}$
6. $\theta = \frac{\pi}{6} + 2\pi k$ or $\theta = \frac{5\pi}{6} + 2\pi k$ or $\theta = \frac{7\pi}{6} + 2\pi k$ or $\theta = \frac{11\pi}{6} + 2\pi k$, where $k \in \mathbb{Z}$
   This may be written more concisely as $\theta = \frac{\pi}{6} + \pi k$ or $\theta = \frac{5\pi}{6} + \pi k$, where $k \in \mathbb{Z}$
7. $\theta = \frac{\pi}{3} + 2\pi k$ or $\theta = \frac{5\pi}{3} + 2\pi k$
8. $\theta = \frac{\pi}{4} + 2\pi k$ or $\theta = \frac{3\pi}{4} + 2\pi k$ or $\theta = \frac{5\pi}{4} + 2\pi k$ or $\theta = \frac{7\pi}{4} + 2\pi k$, where $k \in \mathbb{Z}$
   This can be stated more concisely as $\theta = \frac{\pi}{4} + \frac{\pi}{2}k$, where $k \in \mathbb{Z}$
9. $2\sin^2\theta - 1$ is not equivalent.
   $$\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) = (\cos^2\theta - \sin^2\theta)(1) = \cos^2\theta - \sin^2\theta$$
   $$= \cos^2\theta - (1 - \cos^2\theta) = 2\cos^2\theta - 1$$
10. $\frac{\cos\theta}{\tan\theta - \sec\theta}$ is not equivalent.
   $$\csc\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta = \frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta} = \frac{1 - \sin^2\theta}{\sin\theta} = \frac{(1 + \sin\theta)(1 - \sin\theta)}{\sin\theta} = \frac{1 - \sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$$
11. $\sin^2\theta - \cos^2\theta$ is not equivalent.
   $$(\tan\theta + \cot\theta)^2 = \tan^2\theta + 2\tan\theta\cot\theta + \cot^2\theta = \tan^2\theta + 2\tan\theta\left(\frac{1}{\tan\theta}\right) + \cot^2\theta = \tan^2\theta + 2 + \cot^2\theta$$
   $$= \tan^2\theta + 1 + \cot^2\theta + 1 = \sec^2\theta + \csc^2\theta$$
12. $\cos^2\theta - \sin^2\theta$ is not equivalent.
   $$\frac{\tan\theta}{1 + \tan^2\theta} = \frac{\tan\theta}{\sec^2\theta} = \frac{\sin\theta/\cos\theta}{1/\cos^2\theta} = \frac{\sin\theta}{\cos\theta} \cdot \frac{\cos^2\theta}{1} = \sin\theta\cos\theta = \frac{1}{\csc\theta} \cdot \cos\theta = \frac{\cos\theta}{\csc\theta}$$